CHAPTER 1: PROGRESSION

Arithmetic Progression

$T_n = a + (n − 1) d$

$S_n = \dfrac{n}{2} [2a + (n − 1)d]$

$\quad = \dfrac{n}{2} [a + T_n]$

$S_1 = T_1 = a$

$T_2 = S_2 − S_1$

Example : The 15th term of an A.P. is −86 and the sum of the first 15 terms is −555. Find
(a) the first term and the common difference
(b) the sum of the first 20 terms
(c) the sum from the 12th to the 20th term.

(a) $T_{15} = −86$

$S_n = \dfrac{n}{2} [a + T_n]$

$−555 = \dfrac{15}{2} [a − 86]$

$−74 = a − 86$

$a = 12$

$a + 14d = −86$

$12 + 14d = −86$

$14d = −98$

$d = −7$

(b) $S_{20} = \dfrac{20}{2} [2(12) + 19(−7)]$

$\qquad = 10[24 − 133]$

$\qquad = −1090$

(c) Sum from 12th to 20th term = $S_{20} − S_{11}$

$\qquad = −1090 − \dfrac{11}{2} [24 + 10(−7)]$

$\qquad = −1090 −(−253)$

$\qquad = −837$

Geometric Progression

$T_n = ar^{n-1}$

$S_n = \dfrac{a(1−r^n)}{1−r} = \dfrac{a(r^n −1)}{r−1}$

For $−1 < r < 1$, sum to infinity

$S_\infty = \dfrac{a}{1−r}$

Example: Given the 4th term and the 6th term of a G.P. is 24 and $10\dfrac{2}{3}$ respectively. Find the 8th term given that all the terms are positive.

$ar^3 = 24$ ------------------(1)

$ar^5 = 10\dfrac{2}{3} = \dfrac{32}{3}$ --------(2)

(2) ÷(1) $\dfrac{ar^5}{ar^3} = \dfrac{32}{3} \times \dfrac{1}{24}$

$\qquad\qquad r^2 = \dfrac{4}{9} \qquad r = \pm\dfrac{2}{3}$

Since all the terms are positive, $r = \dfrac{2}{3}$

$a \times \left(\dfrac{2}{3}\right)^3 = 24$

$a = 24 \times \dfrac{27}{8} = 81$

$T_8 = 81\left(\dfrac{2}{3}\right)^7 = 4\dfrac{20}{27}$

Example: Find the least number of terms of the G.P 18, 6, 2, $\dfrac{2}{3}$, ... such that the last term is less than 0.0003. Find the last term.

$a = 18, r = \dfrac{6}{18} = \dfrac{1}{3}$

$T_n < 0.0003$

$18 \times \left(\dfrac{1}{3}\right)^{n-1} < 0.0003$

$\left(\dfrac{1}{3}\right)^{n-1} < \dfrac{0.0003}{18}$

$(n − 1) \log \dfrac{1}{3} < \log \dfrac{0.0003}{18}$

$n − 1 > \dfrac{\log \dfrac{0.0003}{18}}{\log \dfrac{1}{3}}$

[Remember to change the sign as $\log \dfrac{1}{3}$ is negative]

$n − 1 > 10.01$

n > 11.01
∴ $n = 12$.

$$T_{12} = 18 \times \left(\frac{1}{3}\right)^{11} = 0.0001016$$

Example: Express each recurring decimals below as a single fraction in its lowest term.
(a) 0.7777....
(b) 0.151515...

(a) 0.7777...= 0.7 + 0.07 + 0.007 + ..
$a = 0.7$

$$r = \frac{0.07}{0.7} = 0.1$$

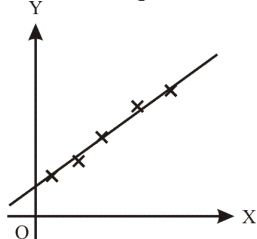$$S_\infty = \frac{a}{1-r} = \frac{0.7}{1-0.1} = \frac{0.7}{0.9} = \frac{7}{9}$$

(b) 0.151515... = 0.15 + 0.0015 + 0.000015....

$a = 0.15$, $r = \frac{0.0015}{0.15} = 0.01$

$$S_\infty = \frac{0.15}{1-0.01} = \frac{0.15}{0.99} = \frac{15}{99} = \frac{5}{33}$$

CHAPTER 2: LINEAR LAW
Characteristic of The Line of Best Fit
1. Passes through as many points as possible.
2. All the other points are as near to the line of best fit as possible.
3. The points which are above and below the line of best fit are equal in number.

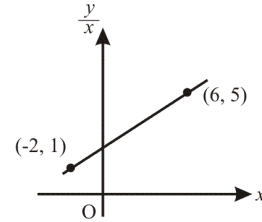### To Convert from Non Linear to Linear Form
To convert to the form Y = mX + c
Example:

Non Linear	Linear	m	c
$y = ab^x$	$\log y = \log b(x) + \log a$	$\log b$	$\log a$
$y = ax^2 + bx$	$\frac{y}{x} = ax + b$	a	b
$y = ax + \frac{b}{x}$	$xy = ax^2 + b$	a	b

Example: The diagram shows the line of best fit by plotting $\frac{y}{x}$ against x.

Find the relation between y and x.
Solution:

m = $\frac{5-1}{6+2} = \frac{1}{2}$, passing through (6, 5)

$5 = \frac{1}{2}(6) + c$ $c = 2$

The equation is $\frac{y}{x} = \frac{1}{2}x + 2$, or

$y = \frac{1}{2}x^2 + 2x$.

Example: The table below shows values of two variables x and y, obtained from an experiment. It is known that y is related to x by the equation $y = ab^x$.

x	1	2	3	4	5	6
y	4.5	6.75	10.1	15.2	22.8	34.1

(a) Explain how a straight line can be obtained from the equation above.
(b) Plot the graph of log y against x by using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 0.2 unit on the y-axis.
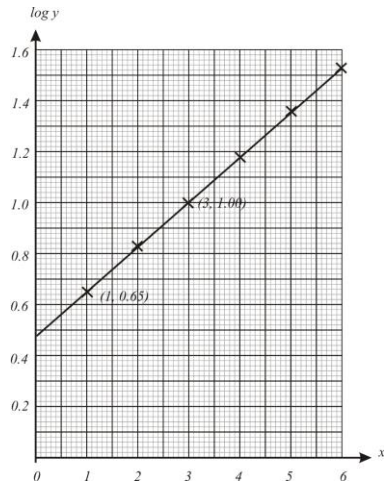(c) From your graph, find the value of a and b.
Solution:
(a) $y = ab^x$
$\log y = \log b(x) + \log a$
By plotting log y against x, a straight line is obtained.

(b)

x	1	2	3	4	5	6
log y	0.65	0.83	1.00	1.18	1.36	1.53

(c) c = log a = 0.48

a = 3

$$\log b = \frac{1.00 - 0.65}{3 - 1} = 0.175$$

b = 1.5

CHAPTER 3: INTEGRATION

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

The area between the graph and the x-axis

$$A = \int y\,dx$$

The area between the graph and the y-axis

$$A = \int x\,dy$$

The volume generated when a shaded region is rotated through 360° about the x-axis

$$V = \int \pi y^2 dx$$

Volume generated when a shaded region is rotated through 360° about the y-axis

$$V = \int \pi x^2 dy$$

Example: Find

(a) $\int 3x^2 + 2x + 3\,dx$

(b) $\int \dfrac{x^4 + 2x}{x^6} dx$

(a) $\int 3x^2 + 2x + 3\,dx$

$$= \frac{3x^3}{3} + \frac{2x^2}{2} + 3x + c$$

$$= x^3 + x^2 + 3x + c$$

(b) $\int \dfrac{x^4 + 2x}{x^6} dx = \int \dfrac{x^4}{x^6} + \dfrac{2x}{x^6} dx$

$$= \int x^{-2} + 2x^{-5} dx$$

$$= \frac{x^{-1}}{-1} + \frac{2x^{-4}}{-4} + c$$

$$= -\frac{1}{x} - \frac{1}{2x^4} + c$$

The Rule of Integration

1. $\displaystyle\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$

2. $\displaystyle\int_a^g f(x) + g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

3. $\displaystyle\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

4. $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

Example:

Given $\displaystyle\int_1^3 f(x)\,dx = 9$, find the value of

(a) $\displaystyle\int_1^3 4f(x)\,dx$

(b) $\displaystyle\int_1^3 [5 + f(x)]\,dx$

(a) $\displaystyle\int_1^3 4f(x)\,dx = 4 \times \int_1^3 f(x)\,dx$

$$= 4 \times 9 = 36$$

(b) $\displaystyle\int_1^3 [5 + f(x)]\,dx = \int_1^3 5\,dx + \int_1^3 f(x)\,dx$
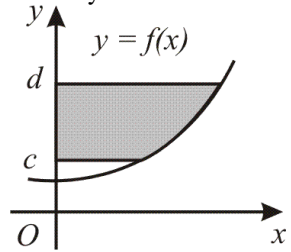
$$= [5x]_1^3 + 9$$

$$= [15 - 5] + 9 = 19$$

Area Below a Graph

1. The area below a graph and bounded by the line x = a, x = b and the x-axis is
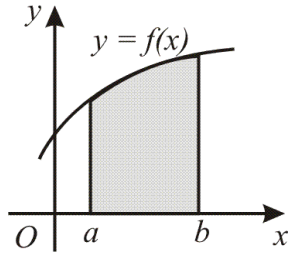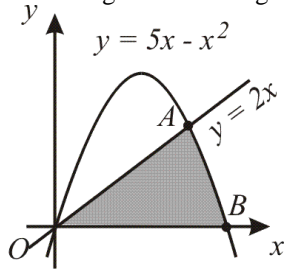
$$A = \int_a^b y\,dx$$

2. Area between the graph and the line y = c, y = d and the y-axis is
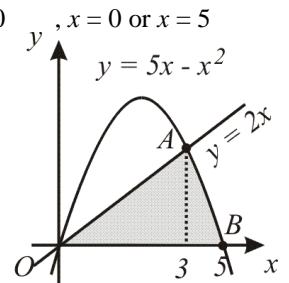

$$A = \int_c^d x\,dy$$

Example:
Given A is the point of intersection between the curve $y = 5x - x^2$ and the line $y = 2x$, find the area of the shaded region in the diagram below.
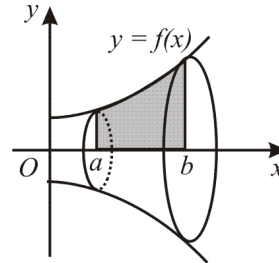

$y = 2x$
$y = 5x - x^2$
$2x = 5x - x^2$
$x^2 - 3x = 0$
$x(x - 3) = 0$
$x = 0$ or 3. $\therefore A(3, 6)$

$5x - x^2 = 0$
$x(5 - x) = 0$, $x = 0$ or $x = 5$

Area under a curve = Area of triangle +

$$\int_3^5 5x - x^2\,dx = \frac{1}{2} \times 3 \times 6 + \left[\frac{5x^2}{2} - \frac{x^3}{3}\right]_3^5$$

$$= 9 + [\frac{125}{2} - \frac{125}{3}] - [\frac{45}{2} - 9]] = 16\frac{1}{3}\ \text{unit}^2$$

Volume of Revolution

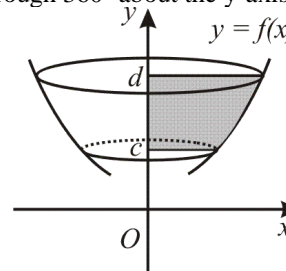Volume generated when a shaded region is revolved through 360° about the x-axis is

$$V = \int_a^b \pi y^2\,dx$$

Volume generated when a shaded region is rotated through 360° about the y-axis is
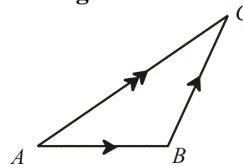

$$V = \int_c^d \pi x^2\,dy$$

CHAPTER 4: VECTORS
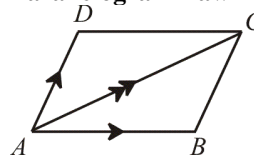**Addition of Two Vectors**
(a)   Triangle Law

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

(b)   Parallelogram Law

$$\overrightarrow{AB}+\overrightarrow{AD}=\overrightarrow{AC}$$

Parallel Vectors

$\overrightarrow{AB}$ is parallel to $\overrightarrow{PQ}$ if $\dfrac{\overrightarrow{AB}}{\overrightarrow{PQ}}=k$, where k is a constant.

If $\dfrac{\overrightarrow{AB}}{\overrightarrow{BC}}=k$, since B is a common point, A, B and C are collinear.

Vector on Cartesian Plane
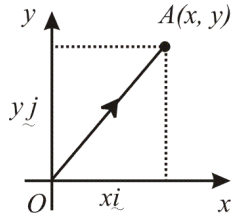
$$\overrightarrow{OA}=x\underset{\sim}{i}+y\underset{\sim}{j}$$

$\left|\overrightarrow{OA}\right|=\sqrt{x^2+y^2}$ = magnitude of vector OA

Unit vector in the direction of

$$\overrightarrow{OA}=\dfrac{x\underset{\sim}{i}+y\underset{\sim}{j}}{\sqrt{x^2+y^2}}$$

Example :
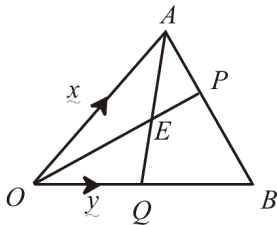
Given $\overrightarrow{OA}=\underset{\sim}{x}$ and $\overrightarrow{OB}=\underset{\sim}{y}$. P is a point on AB such that AP : PB = 1 : 2 and Q is the midpoint of OB. The line OP intersects AQ at the point E.
Given $\overrightarrow{OE}=k\overrightarrow{OP}$ and $\overrightarrow{AE}=h\overrightarrow{AQ}$, where h and k are constants,

(a) find $\overrightarrow{OQ}$ and $\overrightarrow{OP}$ in terms of $\underset{\sim}{x}$ and/or $\underset{\sim}{y}$.

(b) Express $\overrightarrow{OE}$ in terms of
   (i)   k, $\underset{\sim}{x}$ and $\underset{\sim}{y}$,
   (ii) h, $\underset{\sim}{x}$ and $\underset{\sim}{y}$.

(c) Hence, find the value of h and k.

(a) $\overrightarrow{OQ}=\dfrac{1}{2}\overrightarrow{OB}=\dfrac{1}{2}\underset{\sim}{y}$

$$\overrightarrow{OP}=\overrightarrow{OA}+\overrightarrow{AP}=\underset{\sim}{x}+\dfrac{1}{3}\overrightarrow{AB}$$

$$=\underset{\sim}{x}+\dfrac{1}{3}(\underset{\sim}{y}-\underset{\sim}{x})=\dfrac{1}{3}(2\underset{\sim}{x}+\underset{\sim}{y})$$

(b) (i) $\overrightarrow{OE}=k\overrightarrow{OP}=k\times\dfrac{1}{3}(2\underset{\sim}{x}+\underset{\sim}{y})$

$$=\dfrac{2k}{3}\underset{\sim}{x}+\dfrac{k}{3}\underset{\sim}{y}$$

(ii) $\overrightarrow{OE}=\overrightarrow{OA}+\overrightarrow{AE}=\overrightarrow{OA}+h\overrightarrow{AQ}$

$$=\underset{\sim}{x}+h(-\underset{\sim}{x}+\dfrac{1}{2}\underset{\sim}{y})$$

$$=(1-h)\underset{\sim}{x}+\dfrac{h}{2}\underset{\sim}{y}$$

(c) Compare the coefficient of $\underset{\sim}{x}$ and $\underset{\sim}{y}$

$$1-h=\dfrac{2k}{3}\text{ --------------(1)}$$

and $\dfrac{h}{2}=\dfrac{k}{3}$, $h=\dfrac{2k}{3}$ -----(2)

Substitute in (1)

$$1-\dfrac{2k}{3}=\dfrac{2k}{3}$$

$$1=\dfrac{4k}{3}\qquad\therefore k=\dfrac{3}{4}$$

$$h=\dfrac{2k}{3}=\dfrac{2}{3}\times\dfrac{3}{4}=\dfrac{1}{2}.$$

Example :

Given $\overrightarrow{OP}=\begin{pmatrix}3\\-4\end{pmatrix}$ and $\overrightarrow{OQ}=\begin{pmatrix}1\\5\end{pmatrix}$.

(a) Find $\left|\overrightarrow{OP}\right|$

(b) Find the unit vector in the direction of $\overrightarrow{OP}$.

(c) Given $\overrightarrow{OP}=m\overrightarrow{OA}-n\overrightarrow{OQ}$ and A is the point (−2, 7). Find the value of m and n.

(a) $\left|\overrightarrow{OP}\right|=\sqrt{3^2+(-4)^2}=\sqrt{25}=5$

(b) Unit vector in the direction of $\overrightarrow{OP}=$

$$\dfrac{3i-4j}{5}$$

(c) $\overrightarrow{OP}=m\overrightarrow{OA}-n\overrightarrow{OQ}$

$$\begin{pmatrix}3\\-4\end{pmatrix}=m\begin{pmatrix}-2\\7\end{pmatrix}-n\begin{pmatrix}1\\5\end{pmatrix}$$

$-2m - n = 3$ -------(1)
$7m - 5n = -4$ -----(2)
$(1) \times 5$
$-10m - 5n = 15$ ----(3)
$\underline{7m - 5n = -4}$ ----(4)
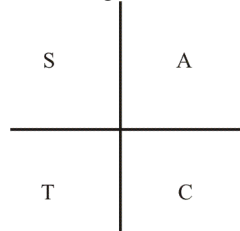$-17m = 19$

$m = -\dfrac{19}{17}$

substitute in (1),

$\dfrac{38}{17} - n = 3$

$n = \dfrac{38}{17} - 3 = -\dfrac{13}{17}$

CHAPTER 5: TRIGONOMETRIC FUNCTION
Angles In The Four Quadrants

S	A
T	C

The Three Trigonometric Functions

Secant θ = sec θ = $\dfrac{1}{\cos\theta}$

Cosecant θ = cosec θ = $\dfrac{1}{\sin\theta}$

Cotangent θ = cot θ = $\dfrac{1}{\tan\theta}$

The Relation Between Trigonometric Functions
Sin θ = cos $(90^o - \theta)$
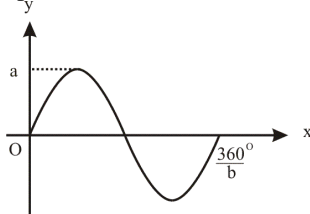Cos θ = sin $(90^o - \theta)$
Tan θ = cot $(90^o - \theta)$

Tan θ = $\dfrac{\sin\theta}{\cos\theta}$

Graphs of Trigonometric Functions
y = asin bx
Amplitude = a
Number of periods = b

The Addition Formulae
Sin $(A \pm B)$ = sin A cos B $\pm$ cos A sin B
Cos $(A \pm B)$ = cos A cos B $\mp$ sin A sin B

Tan $(A \pm B)$ = $\dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

The Double Angle Formulae
Sin 2A = 2 sin A cos A
Cos 2A = $2\cos^2 A - 1$
$\qquad = 1 - 2\sin^2 A$

Tan 2A = $\dfrac{2\tan A}{1 - \tan^2 A}$

CHAPTER 6: PERMUTATION AND COMBINATION
1. Arrangement of n different objects without repetition.

^nP_n = n!

2. Arrangement of r objects from n objects

$^nP_r = \dfrac{n!}{(n-r)!}$

Example : Given the word 'TABLES'. Find
(a) the number of ways to arrange all the letters in the word.
(b) The number of ways of arranging the 6 letters such that the first letter is a vowel.
(c) The number of ways of arranging 4 letters out of the 6 letters such that the last letter is 'S'.

(a) Number of arrangement = $^6P_6$ = 6! = 720
(b) Number of ways of arranging 1 vowel out of 2 = $^2P_1$
Number or ways of arranging the remaining 5 letters = $^5P_5$.
Total arrangement = $^2P_1 \times {}^5P_5$ = 240.

OR:

2	5	4	3	2	1

Total number of ways = $2 \times 5!$ = 240

(c) If the last letter is 'S', the number of ways of arranging 3 letters out of the remaining 5 letters = $^5P_3$ = 60.
OR:

5	4	3	1

Number of ways = $5 \times 4 \times 3 \times 1$ = 60

2. Combination of r objects from n objects

$is\ {}^nC_r = \dfrac{n!}{r!(n-r)!}$

Example: The PTA committee of a school consists of 8 members. The members are elected from 7 parents, 6 teachers and the principal of the school. Find the number of different committees that can be formed if
(a) the principal is one of the member of the committee.
(b) the committee consists of the principal, 3 teachers and 4 parents.
(c) the committee consists of at least 2 teachers.

(a) number of committees = ${}^1C_1 \times {}^{13}C_7 = 1716$
(elect 1 principal, and 7 committee members from 13 = 7 parents and 6 teachers)

(b) number of committees =
${}^1C_1 \times {}^6C_3 \times {}^7C_4 = 700$

(c) number of committees = total number of committees − number of committees with no teacher − number of committees with 1 teacher.
$= {}^{14}C_8 - {}^6C_0 \times {}^8C_8 - {}^6C_1 \times {}^8C_7 = 2954.$

CHAPTER 8: PROBABILITY DISTRIBUTION

1. <u>Binomial Distribution</u>
   The probability of getting r success in n trials where p = probability of success and q = probability of failure
   $P(X = r) = {}^nC_r\, p^r q^{n-r}$

2. Mean , $\mu = np$

3. Standard deviation = $\sqrt{npq}$

Example:
In a survey of a district, it is found that one in every four families possesses computer.
(a) If 6 families are chosen at random, find the probability that at least 4 families possess computers.
(b) If there are 2800 families in the district, calculate the mean and standard deviation for the number of families which possess computer.

(a) $P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6)$
$= {}^6C_4(0.25)^4(0.75)^2 + {}^6C_5(0.25)^5(0.75)$
$\quad + (0.25)^6$
$= 0.03760$

(b) Mean, μ = np = 2800×0.03760
$= 105.3$
Standard deviation = $\sqrt{npq}$ =
$\sqrt{2800 \times 0.3760 \times 0.6240} = 25.63$

2. <u>Normal Distribution</u>
   $Z = \dfrac{X - \mu}{\sigma}$ where
   Z = standard normal score
   X = normal score
   μ = mean
   σ = standard deviation

   $P(Z < a) = 1 - P(Z > a)$
   $P(Z < -a) = P(Z > a)$
   $P(Z > -a) = 1 - P(Z > a)$
   $P(a < Z < b) = P(Z > a) - P(Z > b)$
   $P(-a < Z < b) = 1 - P(Z > a) - P(Z > b)$

Example: The volume of packet drink produced by a factory is normal distributed with a mean of 500 ml and standard deviation of 8 ml. Determine the probability that a packet drink chosen at random has a volume of
(a) more than 510 ml
(b) between 490 ml and 510 ml.

(a) $P(X > 510) = P(Z > \dfrac{510 - 500}{8})$
$= P(Z > 1.25) = 0.1056$

(b) $P(490 < X < 510)$
$= P(\dfrac{490 - 500}{8} < Z < \dfrac{510 - 500}{8})$
$= P(-1.25 < Z < 1.25)$
$= 1 - P(Z > 1.25) - P(Z > 1.25)$
$= 1 - 2(0.1056)$
$= 0.7888$

CHAPTER 9: MOTION ALONG A STRAIGHT LINE

1. <u>Displacement (S)</u>
   Positive displacement $\rightarrow$ particle at the right of O

Negative displacement → particle at the left of O.
Return to O again → s = 0
Maximum/minimum displacement
$$\frac{ds}{dt} = 0$$
Distance travelled in the nth second
= $S_n - S_{n-1}$
Example: Distance travelled in the third second = $S_3 - S_2$

Example: A particle moves along a straight line and its displacement, s meter, from a fixed point O, t seconds after leaving O is given by s = 2t − t^2. Find
(a) the displacement of the particle after 5 seconds,
(b) the time at which it returns to O again.
(c) the distance travelled in the fourth second.

(a) s = 2t − t^2
    t = 5,
    s = 10 − 25 = −15 m

(b) Return to O again → s = 0
    2t − t^2 = 0
    t(2 − t) = 0
    t = 0 or t = 2 second
    ∴ the particle returns to O again when t = 2 s.

(c) Distance travelled in the 4th second = S_4 − S_3 = (8 − 16) − (6 − 9) = − 8 + 3 = − 5 m.
    ∴ the distance travelled in the 4th second is 5 m.

2. Velocity (v)
   Velocity is the rate of change of displacement with respect to time.
   $$v = \frac{ds}{dt}$$
   Initial velocity → the value of v when t = 0
   Instantaneously at rest/change direction of movement → v = 0
   Moves towards the right → v > 0
   Moves towards the left → v < 0
   Maximum/minimum velocity
   $$\rightarrow \frac{dv}{dt} = 0$$
   $$s = \int v\, dt$$
   Distance travelled in the time interval t = a until t = b

(a) If the particle does not stop in the time interval
    $$\text{Distance} = \int_{a}^{b} v\, dt$$
(b) If the particle stops in the time t = c seconds where c is in the interval a → b
    $$\text{Distance} = \left| \int_{a}^{c} v\, dt \right| + \left| \int_{c}^{b} v\, dt \right|$$

Example: A particle moves along a straight line passing through a fixed point O. Its velocity, v m s^{-1}, t seconds after passing through O is given by v = 2t + 3. Find the displacement at the time of 4 second.
$$s = \int v\, dt = \int 2t + 3\, dt = t^2 + 3t + c$$
If the particle passes through O when t = 0, ∴, s = 0 when t = 0. ∴ c = 0
∴ s = t^2 + 3t
When t = 4 s,
s = 16 + 12 = 28 m

Example:
A particle moves along a straight line passing through a fixed point O. Its velocity, v m s^{-1}, t seconds after passing through O is given by v = t^2 + t − 6. Find
(a) the initial velocity of the particle,
(b) the time when the particle is momentarily at rest.

(a) initial velocity → t = 0
    v = − 6 m s^{-1}

(b) momentarily at rest → v = 0
    t^2 + t − 6 = 0
    (t + 3)(t − 2) = 0
    t = −3 or t = 2 s
    Since the time cannot be negative, ∴ t = 2 s.

3. Acceleration (a)
   Acceleration is the rate of change of velocity with respect to time.
   $$a = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$
   Initial acceleration → a when t = 0
   Deceleration → a < 0
   Positive acceleration → velocity increasing
   Negative acceleration → velocity decreasing

Zero acceleration $\rightarrow$ uniform velocity.
Maximum/minimum velocity $\rightarrow$ a = 0

$$v = \int a \, dt$$

Example: A particle moves along a straight line passing through a fixed point O. Its velocity, v m s^{-1}, t seconds after leaving O is given by $v = t^2 - 6t - 7$. Find
(a) the initial acceleration of the particle
(b) the time when the velocity of the particle is maximum.

(a) $a = \dfrac{dv}{dt} = 2t - 6$

      t = 0, a = −6 m s^{-2}

(b) Maximum velocity, a = 0
      2t − 6 = 0
      t = 3 s.

Example : A particle moves along a straight line passing through a fixed point O with a velocity of 5 m s^{-1}. Its acceleration, a ms^{-2} a the time t second after leaving O is given by $a = 2t - 4$. Find the maximum velocity of the particle.

$a = 2t - 4$

$v = \int 2t - 4 \, dt = t^2 - 4t + c$

t = 0, v = 5, $\therefore$ c = 5
$v = t^2 - 4t + 5$
For maximum velocity, a = 0
2t − 4 = 0, t = 2 s.
$v_{max} = 4 - 8 + 5 = 1$ m s^{-1}

CHAPTER 10: LINEAR PROGRAMMING
Steps
1. Form the linear inequalities.
2. Construct the region which satisfies the constraints.
3. Form the optimum equation
     ax + by = k
4. Find the point in the region which gives the maximum or minimum value.
5. Substitute the value of x and y to obtain the optimum value of k.

Example:

School uniform	Time of preparation (minutes)
Shirt	10
Shorts	20

The table above shows the time taken by a tailor to prepare a shirt and a shorts of a school uniform. In a week, the tailor sells x shirts and y shorts. Given that in a week, the number of shirts and shorts sold must be at least 10. The time for preparation is at most 800 minutes. The ratio of the number of shorts to the shirts must be at least 1 : 2.
(a) Write three inequalities other than $x \geq 0$ and $y \geq 0$ which satisfy the constraints above.
(b) By using a scale of 2 cm to 10 units on the x- and y-axes, draw the graph of all the inequalities above. Hence, shade the region R which satisfies the constraints above.
(c) The tailor makes a profit of RM5 and RM3 in selling a shirt and a shorts respectively. Find the maximum profit made by the tailor in a week.

(a) the inequalities are:
     (i) $x + y \geq 10$
     (ii) $10x + 20y \leq 800$
          $x + 2y \leq 80$
     (iii) $\dfrac{y}{x} \geq \dfrac{1}{2}$

          $y \geq \dfrac{1}{2}x$

(b) To draw x + y = 10, x = 0, y = 10 and y = 0, x = 10
     To draw x + 2y = 80
     x = 0, 2y = 80, y = 40
     y = 0, x = 80

     To draw $y = \dfrac{1}{2}x$
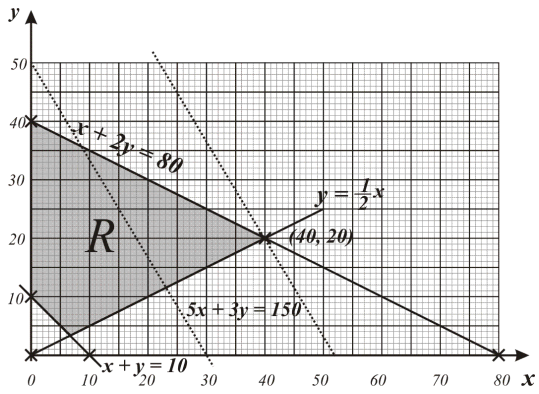
     x = 0, y = 0
     x = 40, y = 20.

(c) Profit , k = 5x + 3y
     Let k = 150
     5x + 3y =150
     x = 0, 3y = 150, y = 50
     y = 0, 5x = 150, x = 30

From the graph, maximum profit is achieved when x = 40 and y = 20.

∴ maximum profit = $5 \times 40 + 3 \times 20 = $ RM 260.

ALL THE BEST FOR YOUR SPM EXAM.