(26) MATRIKS
MATRICES

(a) MATRIKS / MATRIKS SAMA (Matrices / Equal Matrices)

Matriks	Matriks Sama
Contoh : $\begin{pmatrix} 9 & 4 & 1 \\ 7 & 3 & 2 \end{pmatrix}$ ← Baris1, ← Baris2 ↓ ↓ ↓ lajur1 lajur2 lajur3 $\Rightarrow$ $\therefore$ Susunan matriks (order of matrix) $= 2 \times 3$ $\therefore\ a_{13} = 1$ $\therefore\ a_{21} = 7$	$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ -6 & 8 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & h \\ k & 8 \end{pmatrix}$ Jika $\mathbf{A} = \mathbf{B}$, $\therefore\ h = 3,\quad k = -6$

(b) PENAMBAHAN DAN PENOLAKAN MATRIKS (Addition and Subtration of matrices)

Penambahan	Penolakan
• $(a \quad b) + (c \quad d) = (a+c \quad b+d)$	• $(a \quad b) - (c \quad d) = (a-c \quad b-d)$
• $\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a+c \\ b+d \end{pmatrix}$	• $\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a-c \\ b-d \end{pmatrix}$
• $\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$	• $\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a-e & b-f \\ c-g & d-h \end{pmatrix}$

(c) Pendaraban Skalar (Scalar multiplication)

• $k(a \quad b) = (ka \quad kb)$	• $k\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ka \\ kb \end{pmatrix}$	• $k\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kc \\ kc & kd \end{pmatrix}$

Contoh 1 :

$$\begin{pmatrix} 5 & 2 \\ 7 & 1 \end{pmatrix} + 3\begin{pmatrix} 2 & 0 \\ 2 & -3 \end{pmatrix} - \begin{pmatrix} -3 & 1 \\ -2 & 5 \end{pmatrix} = ???$$

$$= \begin{pmatrix} 5 & 2 \\ 7 & 1 \end{pmatrix} + \begin{pmatrix} 6 & 0 \\ 6 & -9 \end{pmatrix} - \begin{pmatrix} -3 & 1 \\ -2 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 5 + 6 - (-3) & 2 + 0 - 1 \\ 7 + 6 - (-2) & 1 + (-9) - 5 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 1 \\ 15 & -13 \end{pmatrix}$$

Contoh 2 :

$2(4 \quad h) + k(1 \quad 2) = (13 \quad 8),\quad h + k = ???$

$(8 \quad 2h) + (k \quad 2k) = (13 \quad 8)$

$8 + k = 13$
$k = 13 - 8$
$k = 5$

$2h + 2k = 8$
$2h + 2(5) = 8$
$2h + 10 = 8$
$2h = -2$
$h = -1$

$\therefore\ h + k = -1 + 5$
$= 4$

(d) PENDARABAN DUA MATRIKS (Multiplication of two matrices)

• $\begin{pmatrix} a & b \end{pmatrix}\begin{pmatrix} c \\ d \end{pmatrix} = (ac+bd)$ **(M 12) (M 21) = (M 11)**	• $\begin{pmatrix} a \\ b \end{pmatrix}\begin{pmatrix} c & d \end{pmatrix} = \begin{pmatrix} ac & ad \\ bc & bd \end{pmatrix}$ **(M 21) (M12) = (M22)**	• $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e \\ f \end{pmatrix} = \begin{pmatrix} ae + bf \\ ce + df \end{pmatrix}$ **(M 22) (M21) = (M21)**
• $\begin{pmatrix} a & b \end{pmatrix}\begin{pmatrix} c & e \\ d & f \end{pmatrix} = \begin{pmatrix} ac + bd & be + bf \end{pmatrix}$ **(M 12) (M22) = (M12)**	• $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$ **(M 22) (M 22) = (M 22)**	
Contoh 1 : $\begin{pmatrix} m \\ 3 \end{pmatrix}(2 \quad -1) = \begin{pmatrix} 6-m & m-4 \\ 6 & -3 \end{pmatrix}, \quad m = ???$ $m(2) = 6 - m$ $2m + m = 6$ $3m = 6$ $m = \frac{6}{3}$ $m = 2$	**Contoh 2 :** $(k \quad 5)\begin{pmatrix} 3 & 0 \\ -k & 1 \end{pmatrix} = (24 \quad 5), \quad k = ???$ $k(3) + 5(-k) = 24$ $3k - 5k = 24$ $-2k = 24$ $k = \frac{24}{-2}$ $k = -12$	

(e) MATRIKS IDENTITI (Identity Matrices), I

- $\mathrm{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\mathrm{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

- **IA = AI = A**

$$\begin{pmatrix} -6 & 0 \\ 5 & 3 \end{pmatrix} P = \begin{pmatrix} -6 & 0 \\ 5 & 3 \end{pmatrix} \Rightarrow \quad P = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(f) MATRIKS SONGSANG (Inverse matrices), A^{-1}

- $\mathrm{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \mathrm{A}^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

- JIKA A tidak mempunyai songsang / A^{-1} tidak wujud $\Rightarrow ad - bc = 0$.

- $\mathrm{AB} = \mathrm{I},\ \mathrm{BA} = \mathrm{I} \Rightarrow \mathrm{A} = \mathrm{B}^{-1},\quad \mathrm{B} = \mathrm{A}^{-1}$

Contoh 1 :

$\mathrm{Q} = \begin{pmatrix} -3 & 5 \\ -1 & 1 \end{pmatrix} \Rightarrow$ matriks songsang Q / Q^{-1} = ???

$$\mathrm{Q}^{-1} = \frac{1}{-3(1) - 5(-1)}\begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -\frac{3}{2} \end{pmatrix}$$

Contoh 2 :

$\begin{pmatrix} 1 & k \\ 2 & 6 \end{pmatrix}$ tidak mempunyai songsangan, k = ???

$$1(6) - k(2) = 0$$

$$6 - 2k = 0$$

$$-2k = -6$$

$$k = \frac{-6}{-2}$$

$$k = 3$$

Contoh 3 :

Matriks songsang $\begin{pmatrix} -\frac{1}{2} & 1 \\ m & n \end{pmatrix}$ adalah $\begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}$.

m = ???, n = ???

$$\begin{pmatrix} -\frac{1}{2} & 1 \\ m & n \end{pmatrix} = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}^{-1}$$

bandingkan

$$= \frac{1}{4(-1) - (-2)(3)}\begin{pmatrix} -1 & 2 \\ -3 & 4 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -1 & 2 \\ -3 & 4 \end{pmatrix}$$

$$\rightarrow = \begin{pmatrix} -\frac{1}{2} & 1 \\ -\frac{3}{2} & 2 \end{pmatrix}$$

$\therefore\ m = -\frac{3}{2},\quad n = 2$

Contoh 4 :

$\mathrm{P} = \begin{pmatrix} 1 & 3 \\ -2 & -1 \end{pmatrix}$, $\mathrm{R} = \frac{1}{m}\begin{pmatrix} -1 & -3 \\ k & 1 \end{pmatrix}$ dan $\mathrm{PR} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

m = ???, k = ???

$$PR = \mathrm{I}$$

$$R = \mathrm{P}^{-1}$$

$$\frac{1}{m}\begin{pmatrix} -1 & -3 \\ k & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ -2 & -1 \end{pmatrix}^{-1}$$

bandingkan

$$= \frac{1}{1(-1) - 3(-2)}\begin{pmatrix} -1 & -3 \\ 2 & 1 \end{pmatrix}$$

$$\rightarrow = \frac{1}{5}\begin{pmatrix} -1 & -3 \\ 2 & 1 \end{pmatrix}$$

$\therefore\ m = 5,\quad k = 2$

(g) PENYELESAIAN MASALAH (Solve simultabeous linear equations using matrices)

Contoh 1 :

$M = \begin{pmatrix} 6 & -3 \\ 5 & -2 \end{pmatrix}$, $M \begin{pmatrix} e \\ f \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \Rightarrow e = ???, f = ???$

$$M \begin{pmatrix} e \\ f \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} e \\ f \end{pmatrix} = M^{-1} \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$= \frac{1}{6(-2) - (-3)(5)} \begin{pmatrix} -2 & 3 \\ -5 & 6 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} -2(0) + 3(2) \\ -5(0) + 6(2) \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} 6 \\ 12 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$\therefore \quad e = 2, \quad f = 4$

Contoh 2 :

$$\begin{matrix} 5x + 2y = 4 \\ -3x - y = -3 \end{matrix} \Rightarrow x = ???, \quad y = ???$$

$$\begin{pmatrix} 5 & 2 \\ -3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ -3 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

$$= \frac{1}{5(-1) - 2(-3)} \begin{pmatrix} -1 & -2 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

$$= \frac{1}{1} \begin{pmatrix} -1(4) + (-2)(-3) \\ 3(4) + 5(-3) \end{pmatrix}$$

$$= 1 \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$\therefore \quad x = 2, \; y = -3$