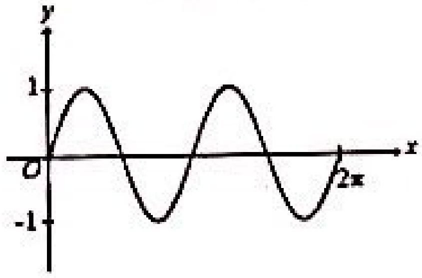
Additional Mathematics Paper 2
SPMRSM 2020

Answer Scheme

No	Solution	Scheme	Sub marks	Marks
1 (a)	LHS: $=\frac{\sec^2 A-\tan^2 A}{\tan A+\cot A}$ $=\frac{1}{\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}}$ $=\frac{1}{\frac{\sin^2 A+\cos^2 A}{\sin A\cos A}}$ $=\frac{\sin A\cos A}{\sin^2 A+\cos^2 A}$ $=\frac{\sin A\cos A}{1}$ $=\frac{\sin 2A}{2}$ = RHS	K1 Use $\sec^2 A-\tan^2 A=1$ or $\tan A=\frac{\sin A}{\cos A}$ or $\cot A=\frac{\cos A}{\sin A}$ or $\sin^2 A+\cos^2 A=1$ or $\sin 2A=2\sin A\cos A$ N1 $\frac{\sin 2A}{2}$ LHS = RHS	2	
(b)	(i) 	Shape of sine graph P1 2 cycles of $0^\circ \leq A \leq 360^\circ$ P1 Max = 1, min = -1 P1		3
	(ii) The possible number of solutions are 2, 4 and 5.	N1 2, 4 and 5 All correct		1

No	Solution	Scheme	Sub marks
4	$y=\frac{3}{2x-2}$ $\frac{dy}{dx}=\frac{-6}{(2x-2)^2}$ $-\frac{3}{2}=\frac{-6}{(2x-2)^2}$ $(2x-2)^2=4$ $x=0,\ x=2$	K1 Find $\frac{dy}{dx}$ K1 Equate ${}^{*}\frac{dy}{dx}=-\frac{3}{2}$ N1 0 and 2	3
	when $x=2$ $y=\frac{3}{2(2)-2}=\frac{3}{2}$ $y-{}^{*}\frac{3}{2}=-\frac{3}{2}(x-{}^{*}2)$ $y=-\frac{3}{2}x+\frac{9}{2}$ $y=-\frac{3}{2}(1)+\frac{9}{2}$ $=3$	K1 Substitute ${}^{*}x$ into $y=\frac{3}{2x-2}$ to find y K1 Use ${}^{*}x$ and ${}^{*}y$ to find the equation of straight line N1 $y=-\frac{3}{2}x+\frac{9}{2}$ N1 $y=-\frac{3}{2}(1)+\frac{9}{2}$ $=3$ Yes, the car will hit the cone.	4

No	Solution	Scheme	Sub marks	Marks
3				
(a)	$\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16}\ldots$ $\frac{1}{4}\div\frac{1}{2}=\frac{1}{2}$ $\frac{1}{2}\left(\frac{1}{2}\right)^{n-1}<0.0001$ $n>13$ $n=14$, Square	K1 Find r $\frac{1}{4}\div\frac{1}{2}$ K1 Use the formula of $T_n=ar^{n-1}$ $\frac{1}{2}\left(\frac{1}{2}\right)^{n-1}$ N1 $n=14$, Square	3	
(b)	$\frac{1}{2},\frac{1}{8},\frac{1}{32},\ldots$ $S_\infty=\frac{\frac{1}{2}}{1-\frac{1}{4}}$ $=\frac{2}{3}$	P1 List the series of rectangles or state the value of $a=\frac{1}{2},r=\frac{1}{4}$ $\frac{1}{2},\frac{1}{8},\frac{1}{32},\ldots$ K1 Use the formula of $S_\infty=\frac{a}{1-r}$ $S_\infty=\frac{\frac{1}{2}}{1-\frac{1}{4}}$ N1 $\frac{2}{3}$	3	6

No	Solution	Scheme	Sub marks
			7
2	$lw=200$	$\boxed{\text{P1}}$	
	$2l+2w=57$	$\boxed{\text{P1}}$	
	$l=\frac{200}{w}$ or $w=\frac{200}{l}$ OR $l=\frac{57-2w}{2}$ or $w=\frac{57-2l}{2}$	P1 Implied	
	$2\left(\frac{200}{w}\right)+2w-57=0$ or $2l+2\left(\frac{200}{l}\right)-57=0$ OR $w\left(\frac{57-2w}{2}\right)-200=0$ or $l\left(\frac{57-2l}{2}\right)-200=0$ $2l^2-57l+400=0$ OR $2w^2-57w+400=0$	K1 Eliminate l or w	
	Factorisation: $(2l-25)(l-16)=0$ OR $(2w-25)(w-16)=0$ OR **Formula:** $l=\frac{-(-57)\pm\sqrt{(-57)^2-4(2)(400)}}{2(2)}$ or $w=\frac{-(-57)\pm\sqrt{(-57)^2-4(2)(400)}}{2(2)}$ Note: a, b and c must correct OR **Completing the square:** $\left(l-\frac{57}{4}\right)^2-\left(-\frac{57}{4}\right)^2+200=0$ or $\left(w-\frac{57}{4}\right)^2-\left(-\frac{57}{4}\right)^2+200=0$	K1 Solve quadratic equation using factorization or formulae or completing the $\boxed{\text{N1}}$ $l=16$ or $w=12.5$ N1 $width=12.5$ and $length=16$ Note: (i) OW -1 if method of solving quadratic equation not shown (ii) SS – 1 if improper factorization is shown (iii) l and w can be any letters/ symbols	
	$l=16$ and $w=12.5$		

No	Solution	Scheme	Sub marks	Marks
5 (a)	$4^2=\frac{260}{5}-\bar{x}^2$ $16-\frac{260}{5}=\bar{x}^2$ $\bar{x}^2=36$ $\bar{x}=6$	K1 Use formula of variance $4^2=\frac{260}{5}-\bar{x}^2$ N1 6	2	
(b)	$\frac{\sum x_{Omega}}{5}=6$ $\sum x_{Omega}=30$ $\frac{\sum x_{Gamma}}{4}=6$ $\sum x_{Gamma}=24$ $25=\frac{\sum x^2_{Gamma}}{4}-6^2$ $\sigma^2_{Omega+Gamma}=\frac{(260+244)}{5+4}-\left[\frac{(30+24)}{5+4}\right]^2$ $=20$	K1 Use formula of mean OR Combine mean $\frac{\sum x_{Omega}}{5}=6$ or $\frac{\sum x_{Gamma}}{4}=6$ OR $\bar{x}=6$ (implied) K1 Use formula of variance $25=\frac{\sum x^2_{Gamma}}{4}-6^2$ K1 Combine variance $\sigma^2_{Omega+Gamma}=\frac{(260+244)}{5+4}-\left[\frac{(30+24)}{5+4}\right]^2$ N1 20	4	6

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No	Solution	Scheme	Sub marks	M
6 (a)	$-3m_2=-1$ $m_2=\frac{1}{3}$ $y-4=\frac{1}{3}(x+3)$ $y=\frac{1}{3}x+5$	K1 Use $m_1m_2=-1$ $-3m_2=-1$ K1 Use m and coordinates $C(-3,4)$ to form equation $y-4=\frac{1}{3}(x+3)$ N1 $y=\frac{1}{3}x+5$	3	
(b)	(i) Midpoint $AB=\left(\frac{2+4}{2},\frac{9+3}{2}\right)$ $=(3,6)$ $\sqrt{(x-3)^2+(y-6)^2}=5$ $x^2+y^2-6x-12y+20=0$	K1 Use formula midpoint $\left(\frac{2+4}{2},\frac{9+3}{2}\right)$ K1 Use distance formula and equate to 5 $\sqrt{(x-3)^2+(y-6)^2}=5$ N1 $x^2+y^2-6x-12y+20=0$	3	
	(ii) $x^2+(0)^2-6x-12(0)+20=0$ $(-6)^2-4(1)(20)$ $-44<0$	K1 Substitute $y=0$ into equation of locus $x^2+(0)^2-6x-12(0)+20=0$ N1 $-44<0$	2	8

No	Solution	Scheme	Sub marks	Marks
7 (a)	$y=\frac{6x^2}{2}+c$ $15=\frac{6(^{*}15)^2}{2}+c$ c = 12 $y=3x^2+12$	K1 **Integrate correctly** $y=\frac{6x^2}{2}+c$ K1 Substitute $(1,^{*}15)$ in the equation $15=\frac{6(^{*}15)^2}{2}+c$ N1 $y=3x^2+12$	3	
(b)	$A_1=\int_0^1(3x^2+12)dx=13$ $A_2=\frac{1}{2}(4+15)(1)$ *or* $\int_0^1(11x+4)dx$ $=13-\frac{19}{2}$ $=\frac{7}{2}$ or 3·5	K1 Find A_1 $\int_0^1(3x^2+12)dx$ K1 Find A_2 $\frac{1}{2}(4+15)(1)$ or $\int_0^1(11x+4)dx$ K1 $^{*}A_1-{}^{*}A_2$ $13-\frac{19}{2}$ N1 $\frac{7}{2}$ or 3.5	4	10
(c)	$V=\pi\int_{12}^{17}\left(\frac{y}{3}-4\right)dy$ $\pi\left\|\frac{y^2}{6}-4y\right\|_{12}^{17}$ $\pi\left(\frac{17^2}{6}-4(17)\right)-\left(\frac{12^2}{6}-4(12)\right)\Big\|_{12}^{17}$ $\frac{25}{6}\pi$ or $4{\cdot}167\pi$	K1 Integrate V $V=\pi\int_{12}\left(\frac{y}{3}-4\right)dy$ K1 Use correct limit $^{*}\pi\left\|\frac{y^2}{6}-4y\right\|_{12}^{17}$ N1 $\frac{25}{6}\pi$ or $4{\cdot}167\pi$	3	

No	Solution	Scheme	Sub marks	Marks
9 (a)	$\angle RPS = 60^\circ$ $60^\circ \times \frac{3\cdot142}{180^\circ} = 1\cdot047 rad$ $r = \frac{14\cdot66}{1\cdot047^*}$ $= 14$ cm radius of semicircle $OPQS = 14 \div 2 = 7$	P1 Seen 60° K1 Use formula $s = r\theta$ 66 = r(1.04734) N1 7	3	10
(b)	$^*7(3.142) + 14 + 14.66$ 50.65	K1 Use $s = r\theta$ and find the perimeter $^*7(3.142) + 14 + 14.66$ N1 50.65		
(c)	Area $A = \frac{1}{2} \times {}^*14^2 \times 1\cdot047 - \frac{1}{2} \times {}^*7^2 \times 2.095 - \frac{1}{2} \times {}^*7^2 \sin^* 60^\circ$ Area $B = \frac{1}{2} \times {}^*7^2 \times 1\cdot047 - \frac{1}{2} \times {}^*7^2 \sin^* 60^\circ$ Area $A = 30\cdot085$ Area $B = 4\cdot434$ Area shaded region = Area A + Area B $\left(\frac{1}{2} \times 1\cdot047 \times {}^*14^2 - \frac{1}{2} \times 2\cdot095 \times {}^*7^2 - \frac{1}{2} \times {}^*7^2 \sin^* 60^\circ\right)$ 102.606 − 51.3125 21.2176 30.0855 $+ \left(\frac{1}{2} \times 1\cdot047 \times {}^*7^2 - \frac{1}{2} \times {}^*7^2 \sin^* 60^\circ\right)$ 25.6515 21.2175 46.8871 $34\cdot49 \leftrightarrow 34\cdot54$	K1 Use formula to find the area of sector $\frac{1}{2} r^2 \theta$ $\frac{1}{2} \times {}^*14^2 \times 1\cdot047$ or $\frac{1}{2} \times {}^*7^2 \times 2\cdot095$ or $\frac{1}{2} \times {}^*7^2 \times 1\cdot047$ K1 Find the area of triangle $\frac{1}{2} r^2 \sin\theta$ or $\frac{1}{2} \times \text{base} \times \text{height}$ $\frac{1}{2} \times 7^2 \sin^* 60^\circ$ or $\frac{1}{2} \times \frac{7}{2} \times \frac{7\sqrt{3}}{2}$ K1 Find area of A or B K1 Area A + Area B N1 $34\cdot49 \leftrightarrow 34\cdot54$ *Accept value in radian	7	

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No		Solution	Scheme	Sub marks	
8 (a)	(i)	$P\left(\frac{36.2-40.5}{5} < Z < \frac{45.0-40.5}{5}\right)$ $= P(-0.86 < Z < 0.9)$ $= 0{\cdot}621$ // $0{\cdot}6211$	K1 Use $Z = \frac{X-\mu}{\sigma}$ $\frac{36.2-40.5}{5}$ or $\frac{45.0-40.5}{5}$ N1 0·621 // 0·6211	2	10
	(ii)	$P(X < k) = \frac{63}{750}$ $= 0.084$ $P\left(Z < \frac{k-40.5}{5}\right) = 0.084$ $\frac{k-40.5}{5} = -1.378$ $k = 33{\cdot}61$	P1 $Z = \{-\}1.378$ K1 Use of *Z and equate to $\frac{k-40.5}{5}$ $\frac{k-40.5}{5} = -1.378$ N1 33·61	3	
(b)	(i)	$P(X=5) = {}^5C_5(0.8)^5(0.2)^0$ $= 0{\cdot}3277$	K1 Use ${}^nC_r p^r q^{n-r}$ ${}^5C_5(0.8)^5(0.2)^0$ N1 0·3277	2	
	(ii)	$1 - {}^nC_0(0.8)^0(0.2)^n > 0.9$ $n \lg 0.2 < \lg 0.1$ $n > 1.431$ $n = 2$	K1 $P(X \geq 1) > 0{\cdot}9$ K1 $1 - {}^nC_0(0.8)^0(0.2)^n > 0.9$ N1 2	3	

<table>
<tr><th>No</th><th>Solution</th><th>Scheme</th><th>Sub marks</th><th>Marks</th></tr>
<tr><td>11
(a)</td><td>

$\log_{10} x$	0·30	0·60	0·90	1·20	1·38	1·46
y	4·8	6·3	7·8	9·3	10·1	10·6

</td><td>N1 Note: At least 2 d.p</td><td></td><td></td></tr>
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(b)</td><td>Plot y against $\log_{10} x$

*6 points plotted correctly

Line of best fit
(All *points correctly plotted)

$y=\frac{2\log_{10} x}{\log_{10} a}+\frac{\log_{10} b}{\log_{10} a}$

(i) $\log_{10} x = 1.14$
$x = 13.8$

(ii) $m = 5$
$5=\frac{2}{\log_{10} a}$
$\log_{10} a = 0.4$

$a = 2{\cdot}512$
$a = 2{\cdot}51 \leftrightarrow 2{\cdot}55$

(i) $c = 3{\cdot}3$
$3{\cdot}3=\frac{\log_{10} b}{0{\cdot}4}$
$\log_{10} b = 1.32$

$b = 20{\cdot}89$
$b = 20{\cdot}89 \leftrightarrow 21{\cdot}71$</td><td>K1

N1

N1

P1 $y=\frac{2\log_{10} x}{\log_{10} a}+\frac{\log_{10} b}{\log_{10} a}$

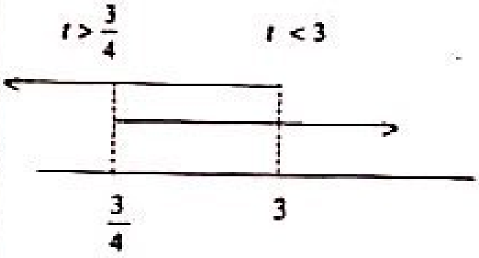
N1 $x = 13{\cdot}8$

Use *$m=\frac{2}{\log_{10} a}$
K1
N1
$a = 2{\cdot}51 \leftrightarrow 2{\cdot}55$

K1 Use *$c=\frac{\log_{10} b}{\log_{10} a}$

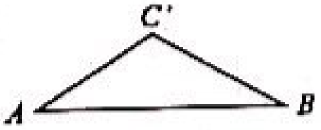
N1 $b = 20{\cdot}89 \leftrightarrow 21{\cdot}71$</td><td>

10</td><td>10</td></tr>
</table>

No	Solution	Scheme	Sub marks
12 (a)	(i) -3	N1 -3	3
	(ii) $V_B = 0$ $6-2t=0$ $t=3$ $S_{max} = 6(^*3)-(^*3)^2$ $= 9$	K1 Differentiate & equate to 0 $6 - 2t = 0$ K1 Substitute $^*t = 3$ into s $S_{max} = 6(^*3)-(^*3)^2$ N1 9	
	(iii) $4t-3>0$ and $6-2t>0$ or $4t-3<0$ and $6-2t<0$  $t>\frac{3}{4}$ $t<3$ $\frac{3}{4}$ 3 $\frac{3}{4}<t<3$	K1 Use $V_A > 0$ and $^*V_B > 0$ or $V_A < 0$ and $^*V_B < 0$ N1 $\frac{3}{4}<t<3$	2
(b)	$S_A = S_B$ $2t^2-3t=6t-t^2$ $3t(t-3)=0$ $t=3$ Total distance particle A: $t=0, s=0$ $t=\frac{3}{4}, s=-\frac{9}{8}$ $t=3, s=9$ $\frac{45}{4}$ or $11\frac{1}{4}$ or $11\cdot25$	K1 Find S_A and equate to S_B K1 Find S_3 or $s_{\frac{3}{4}}$ $S_{\frac{3}{4}} = 2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right)$ or $S_3 = 2(3)^2 - 3(3)$ K1 $\frac{9}{8}+\frac{9}{8}+9$ N1 $\frac{45}{4}$ or $11\frac{1}{4}$ or $11\cdot25$	4

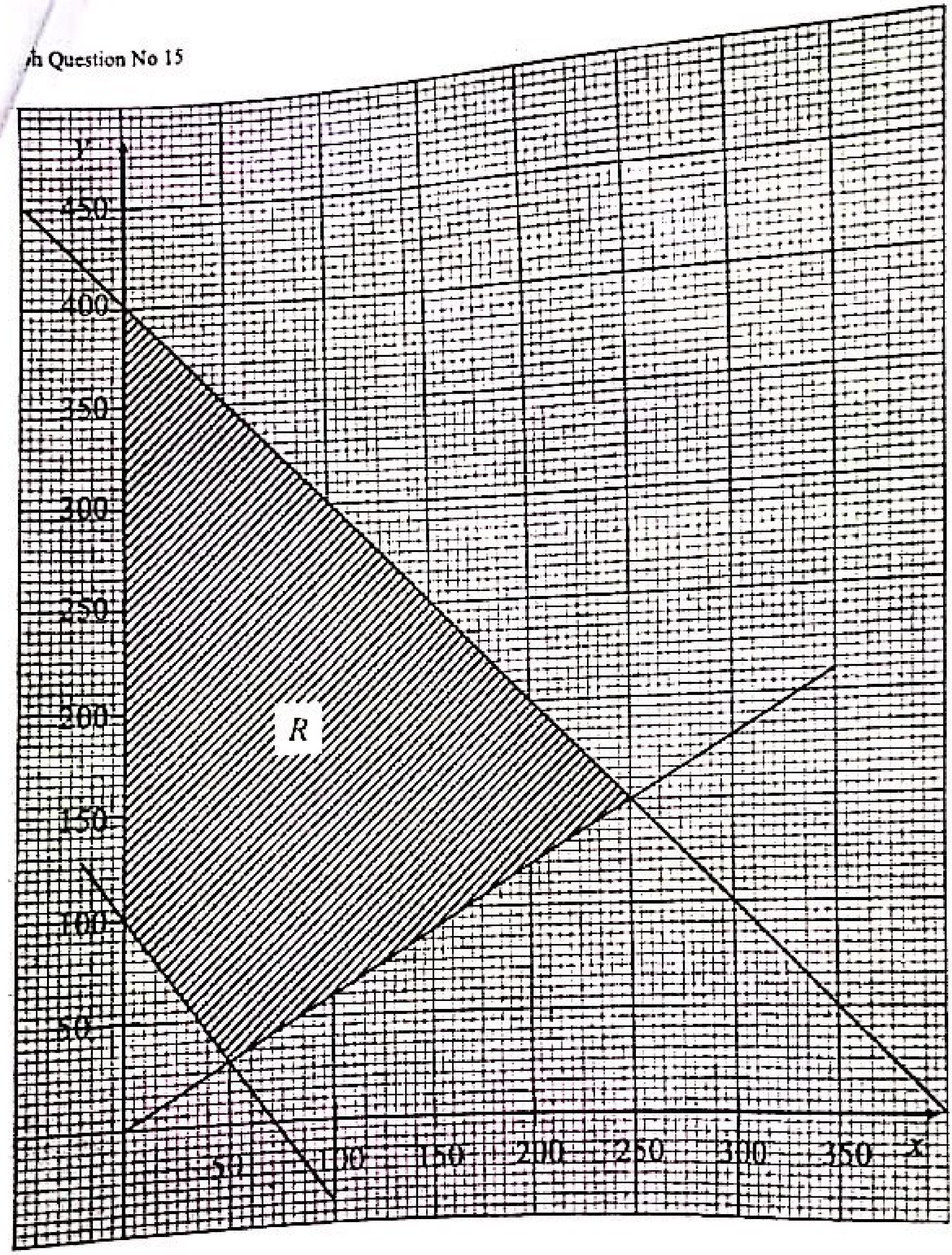
No Solution

13 (a) AC^2

No	Solution	Scheme	Sub marks	Marks
13 (a)	$AC^2 = 4\cdot9^2 + 4^2 - \underbrace{2(4\cdot9)(4)\cos 38^\circ}_{30.89}$ $AC = 3\cdot02$ $\frac{\sin\angle ACB}{4} = \frac{\sin 38^\circ}{{}^*3\cdot02}$ $\angle ACB = 54\cdot63^\circ$	Use cosine rule K1 N1 $AC=3.02$ Use sine rule K1 N1 $\angle ACB = 54.63^\circ$	4	
(b)	(i)  $\angle AC'B = 125\cdot37^\circ$	P1 Sketch $\angle AC'B$ with obtuse angle N1 $125\cdot37^\circ$	2	
	(ii) Area $\Delta AC'B = \frac{1}{2}({}^*3\cdot02)(4)\sin{}^*16\cdot63$ $= 1\cdot729$	K1 Use $\frac{1}{2}ab\sin c$ N1 $1\cdot729$	2	
	(iii) Shortest distance of C' to AB $\frac{1}{2}(4)h = {}^*1\cdot729$ $h = 0\cdot8643$	K1 Use area of triangle or trigonometry N1 $0\cdot8643$	2	
				10

No	Solution	Scheme	Sub marks	
10 (a)	(i) $\overline{SU}=2\underline{a}+3\underline{b}$ (ii) $\overline{TV}=-2\underline{a}+\underline{b}$	N1 $2\underline{a}+3\underline{b}$ N1 $-2\underline{a}+\underline{b}$	2	
(b)	$\overline{SW}=a\overline{SU}$ $=h(2\underline{a}+3\underline{b})$ $\overline{SW}=\overline{ST}+\overline{TW}$ $=2\underline{a}+k(-2\underline{a}+\underline{b})$ $=(2-2k)\underline{a}+k\underline{b}$ $h(2\underline{a}+3\underline{b})=(2-2k)\underline{a}+k\underline{b}$ $2h=2-2k$ $3h=k$ $k=\frac{3}{4},\ h=\frac{1}{4}$	P1 $2h\underline{a}+3h\underline{b}$ K1 Use triangle law N1 $(2-2k)\underline{a}+k\underline{b}$ K1 Equate the coefficient of $\underline{a}$ and $\underline{b}$ N1 $k=\frac{3}{4}$ or $h=\frac{1}{4}$ N1 $h=\frac{1}{4}$ or $k=\frac{3}{4}$	6	
(c)	$\overline{XT}=\overline{XS}+\overline{ST}$ $=-\frac{4}{3}\underline{a}+\underline{b}+2\underline{a}$ $=\frac{2}{3}\underline{a}+\underline{b}$ $=\frac{1}{3}(2\underline{a}+3\underline{b})$ $\overline{XT}=\frac{1}{3}\overline{SU}$ or $\overline{SU}=3\overline{XT}$	K1 $-\frac{4}{3}\underline{a}+\underline{b}+2\underline{a}$ N1 $\overline{XT}=\frac{1}{3}\overline{SU}$ or $\overline{SU}=3\overline{XT}$	2	
				10

h Question No 15



No	Solution	Scheme	Sub marks	Marks
15				
(a)	$x + y \leq 400$ $5y \geq 3x$ $70x + 50y \geq 5\ 000$	P1 P1 P1	3	
(b)	Refer to the graph	K1 Draw correctly at least one straight line from *inequality which involves x and y N1 All three *straight lines are correct Note: **Accept dotted lines** N1 Region shaded correctly	3	
(c)	(i) Maximum fees 70(250) + 50(150) RM 25 000	K1 Use $70x + 50y$ for any point in the *shaded region N1 25 000	2	
	(ii) 30 + 50 80	K1 Use 50 + 30 from the *shaded region N1 80	2	
		Note: SS – 1 once if in (a) i) the symbol '=' is not used at all ii) more than 3 inequalities given (b) i) does not use given scale ii) axes interchanged iii) not using graph paper		10

No	Solution	Scheme	Sub marks	M
14 (a)	(i) $\frac{57.20}{p_{17}}\times 100 = 104$ $p_{17} = 55$	K1 Use $\frac{P_1}{P_0}\times 100$ N1 $p_{17} = 55$	2	
	(ii) $\frac{Q_{19}}{Q_{15}} = \frac{Q_{19}}{Q_{17}}\times\frac{Q_{17}}{Q_{15}}\times 100$ $= \frac{110\times 4x}{100}$ $= 4.4x$	K1 $\frac{Q_{19}}{Q_{15}} = \frac{Q_{19}}{Q_{17}}\times\frac{Q_{17}}{Q_{15}}\times 100$ N1 $4.4x$	2	
(b)	(i) $x + y = 40$ $\frac{120(y) + 4x(20) + 130(x) + 104(40)}{100} = 116\cdot 6$ $120y + 210x + 4160 = 11660$ $120y + 210x = 7500$ $120y + 210(40 - y) = 7500$ $y = 10$ $x = 40 - 10$ $x = 30$	P1 K1 Use $\frac{\Sigma I w}{\Sigma w} = 116\cdot 6$ N1 $y = 10$ or $x = 30$ N1 $x = 30$ or $y = 10$	4	
	(ii) $\frac{110\times I_{19,18}}{100} = 116\cdot 6$ $I_{19,18} = 106$	K1 $\frac{110\times I_{19,18}}{100} = 116\cdot 6$ N1 106	2	10

Graph Question No 11